Statistical tests - Introduction

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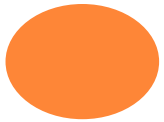
OBJECTIVES

- Define what are the statistical tests
- Explain how a statistical test is performed
- Define the types of errors in statistical tests
- Define statistical significance
- Know the specific language!
- Assess the relationship between two categorical variables using a chi-square test of independence.

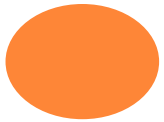


HYPOTHESIS

- The formulation of new hypotheses (or models or theories) is one of the most important aspects of scientific research. These hypotheses are trying to describe or explain real phenomena.

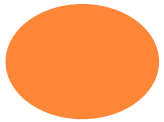
Population - Sample

- When a statistic investigation is carried out on a sample, any result obtained has a relative value.
- We can not even say with certainty what is the difference between the observed data in the sample and the data in population, since the state of the population is usually unknown.



STATISTICAL HYPOTHESIS

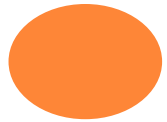
- Therefore, referring to the populations studied starting from the samples, the researchers can not state judgments with certainty, but can express some assumptions which are called **hypotheses**.
- **Statistical tests** will evaluate the statistical hypotheses. After the test, the statistical hypotheses will be accepted or rejected.



How to test a hypothesis

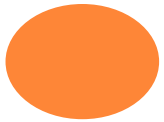
- Formulating scientific hypothesis
  Formulating statistical hypothesis
- Testing
  - Drawing a sample from the target population
  - Checking whether the sample supports the hypothesis
- Repeating the test

- **Test the statistical hypothesis using the appropriate statistical test**



The model for the statistical tests

- Statistical hypothesis testing is done by comparing two competing hypotheses :
  - Null hypothesis - the model that the researcher wants to replace
  - Alternative hypothesis - the new model to replace the null hypothesis



The goal of the statistical test

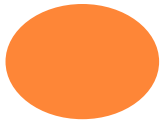
- The goal of the statistical test is to reject the null hypothesis

- The result of the test:
  - Reject the null hypothesis and accept the alternative one with high confidence

  - Can't reject the null hypothesis, can't demonstrate the alternative one

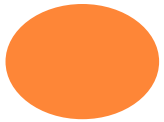
The steps of the statistical test

- **Define the statistical hypotheses**
- **Select the right parameter**
- **Select the level of significance**
- **Select the critical region**
- **Compute the value of the parameter on real data**
- **Decide the result of the test**



Formulation of problems in terms of statistical hypothesis

- Define the null hypothesis - the test is conducted under the assumption that the null hypothesis is true
- Define the alternative hypothesis - also called a working hypothesis – contradicts the null hypothesis



THE PARAMETER

- The statistical parameter of the test expressed in some form the difference between the compared elements

- Is specific for any statistical test type

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$s^2 = \frac{\sum_{i=1}^{n_1}(x_i - \bar{x}_1)^2 + \sum_{j=1}^{n_2}(x_j - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

The level of significance

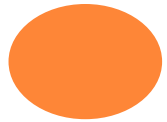
- The level of significance is the size of the risk for errors which the researcher is willing to accept and to assume
- Usually, the level of significance is between 1 şi 5%
- The confidence level = 1-level of significance

The critical region

- The critical region is an interval or a union of intervals
- It may vary from test to test
- It depends by the level of the significance

- $(-\infty, -Z_{\alpha}]$ or $[Z_{\alpha}, +\infty)$
- $[H^2_{\alpha}, +\infty)$
- $(-\infty, -T_{\alpha}] \cup [T_{\alpha}, +\infty)$

- $(-\infty, -1{,}96]$ – CR1
- $(1{,}96, +\infty)$ – CR2



The calculation of the parameter on the research data

- The researchers will compute the value of the parameter using the research data

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$s^2 = \frac{\sum_{i=1}^{n_1}(x_i - \bar{x}_1)^2 + \sum_{j=1}^{n_2}(x_j - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

t = -1,58 for one set of data – DATASET 1

t=6,61 for other set of data – DATASET 2

Decision with critical region

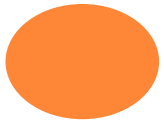
- H_0 will be rejected if the value of the parameter is in the critical region. In this situation, alternative hypothesis is accepted, assuming a risk for error $<\alpha$ – type I error
- If the parameter is outside the critical region, the null hypothesis can not be rejected, assuming a risk for error β – type II error

Dataset 1, CR 1

Significance level = 0,05; Confidence level 95%
CR = (-∞, -1,96] t=-1,58
We cannot demonstrate the alternative hypothesis

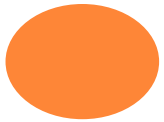
Dataset 2, cr 2

Significance level = 0,05; Confidence level 95%
CR = (1,96, + ∞) t=6,61
With 95% confidence we can accept the alternative hypothesis



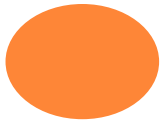
Conclusion

- Statistical tests are used for testing the statistical hypotheses
- We must define the null and alternative hypothesis – the goal of the test is to reject the null hypothesis
- The statistical test does not allow us to accept the null hypotheses – we can or cannot reject it
- By rejecting the null hypothesis, we can accept the alternative one with a specific level of confidence



Remember!

- When you are investigating samples, you must use statistical tests!
- Define the null and the alternative hypotheses
- The null hypothesis starts with “There is NO”
- Don’t worry about the formulas – we have computers!
- You are never allowed to accept the null hypothesis. Only the alternative one could be accepted!
- All the time, the statistical test ends with a conclusion
- The conclusion could be “We reject the null hypothesis and, with 95% of confidence, we accept the alternative one so … “ or “We cannot reject the null hypothesis and we cannot demonstrate that …”

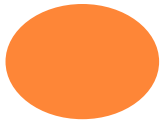


Language

- By rejecting the null hypothesis, the researcher states that the observed results are not due to chance = the results are significant
- When the null hypothesis is not rejected, researcher states that the observed differences are due to chance and the results are not significant

Statistical significance

- P – Probability
- Theoretical value for the minimal level of significance for which the null hypotheses can be rejected
- It is specific to statistical software



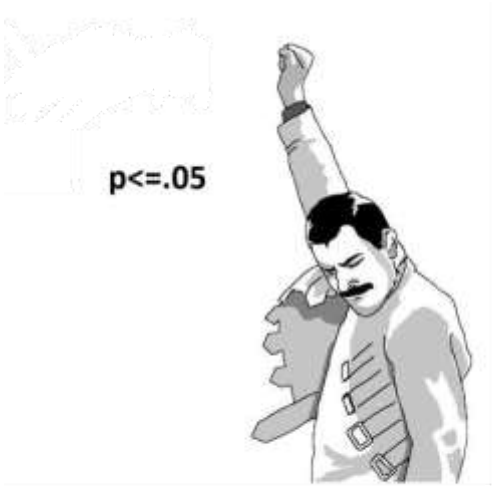
The meaning of p

- $0{,}01 \leq p \leq 0{,}05$ – significant results
- $0{,}001 \leq p < 0{,}01$ – high significant results
- $p < 0{,}001$ – very high significant results
- $p > 0{,}05$ – non-significant results
- $0{,}05 < p \leq 0{,}1$ – tendency

The result of the test using the statistical significance

- The null hypothesis can be rejected if the results are at least significant

- If the result of the test are not significant the null hypothesis can not be rejected

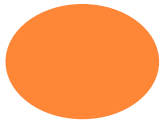


Errors

		Truth	
		H_0 True	H_0 False
Result	H_0 rejected	Type I error (α)	Correct
	H_0 not rejcted	Correct	Type II error (β)

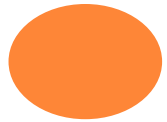
Type I error

- = H_0 is rejected but is true
- We concluded that there are real differences although they are due to chance
- We conclude that a treatment if effective based on a misinterpretation



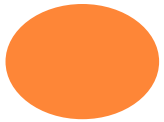
Type II Error

- = H_0 is not rejected even that it is false
- We concluded that the observed differences are due to chance even when they arise due to differences between samples
- We could abandon a new treatment or a new research direction
- "Power of the test" = 1-**β**



Conclusion

- The decision of the statistical test can be made based on the position of the test parameter relative to the critical region or based on the interpretation of the value p – both must be the same
- For any decision, there is a possible error (type I or type II)

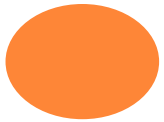


Descriptive statistics parameters

- Mean
- Median
- Variance, etc.

- Frequency

- Difference
- Relationship
- Independence
- Normality
- Correlation, etc.



Chi square test

- Is used to test the independence (association) between **qualitative** variables

- The null hypothesis: there is no association between variables
- The alternative hypothesis: there is association between variables

Chi square by example – dichotomies variables

- We are searching for hypothetical association between smoking and lung cancer. For that, from oncology department records, we select a group of 160 people who were diagnosed with lung cancer. We also select a control group of 240 persons which were never diagnosed with lung cancer. The study of this sample leads us to the following contingency table:

	Cancer present	Cancer absent	Total
Smoking yes	80	50	130
Smoking No	80	190	270
Total	160	240	400

The hypotheses, level of significance, critical region

- H0: there is no association between smoking and lung cancer
- H1: there is association between smoking and lung cancer

- We'll use a level of significance of 5%
- Critical region for 5% level of significance is [3.84, +∞)

THE PARAMETER OF THE TEST

- The test consists by the comparation of the observed contingency table with the theoretical contingency table. The theoretical contingency table contains the theoretical distribution of the data when null hypothesis should be true

	Cancer present	Cancer absent	Total
Smoking yes	a^o	b^o	a^o+b^o
Smoking No	c^o	d^o	c^o+d^o
Total	a^o+c^o	b^o+d^o	$a^o+b^o+c^o+d^o$

Observed contingency table

	Cancer present	Cancer absent	Total
Smoking yes	a^t	b^t	a^t+b^t
Smoking No	c^t	d^t	c^t+d^t
Total	a^t+c^t	b^t+d^t	$a^t+b^t+c^t+d^t$

Theoretical contingency table

Computing the theoretical table

- The null hypotheses change the distribution of the data in the contingency table, but the totals will remain the same

	Cancer present	Cancer absent	Total
Smoking yes	$(a^o+c^o) * (a^o+b^o) / (a^o+b^o+c^o+d^o)$	$(b^o+d^o) * (a^o+b^o) / (a^o+b^o+c^o+d^o)$	a^o+b^o
Smoking No	$(a^o+c^o) * (c^o+d^o) / (a^o+b^o+c^o+d^o)$	$(b^o+d^o) * (c^o+d^o) / (a^o+b^o+c^o+d^o)$	c^o+d^o
Total	a^o+c^o	b^o+d^o	$a^o+b^o+c^o+d^o$

$$a^t=(a^o+c^o) * (a^o+b^o) / (a^o+b^o+c^o+d^o)$$

$$b^t=(b^o+d^o) * (a^o+b^o) / (a^o+b^o+c^o+d^o)$$

$$c^t=(a^o+c^o) * (c^o+d^o) / (a^o+b^o+c^o+d^o)$$

$$d^t=(b^o+d^o) * (c^o+d^o) / (a^o+b^o+c^o+d^o)$$

COMPUTE THE THEORETICAL TABLE

	Cancer present	Cancer absent	Total
Smoking yes	$\frac{160 * 130}{400}$	$\frac{240 * 130}{400}$	130
Smoking No	$\frac{160 * 270}{400}$	$\frac{270 * 240}{400}$	270
Total	160	240	400

	Cancer present	Cancer absent	Total
Smoking yes	52	78	130
Smoking No	108	162	270
Total	160	240	400

CHI SQUARE VALUE

	Cancer present	Cancer absent	Total
Smoking yes	a^o	b^o	a^o+b^o
Smoking No	c^o	d^o	c^o+d^o
Total	a^o+c^o	b^o+d^o	$a^o+b^o+c^o+d^o$

	Cancer present	Cancer absent	Total
Smoking yes	a^t	b^t	a^t+b^t
Smoking No	c^t	d^t	c^t+d^t
Total	a^t+c^t	b^t+d^t	$a^t+b^t+c^t+d^t$

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Total	160	240	400

	Cancer present	Cancer absent	Total
Smoking yes	52	78	130
Smoking No	108	162	270
Total	160	240	400

$$Chi\ square = \frac{(a^o - a^t)^2}{a^t} + \frac{(b^o - b^t)^2}{b^t} + \frac{(c^o - c^t)^2}{c^t} + \frac{(d^o - d^t)^2}{d^t}$$

$$Chi\ square = \frac{(80 - 52)^2}{52} + \frac{(50 - 78)^2}{78} + \frac{(80 - 108)^2}{108} + \frac{(190 - 162)^2}{162} = 37.2$$

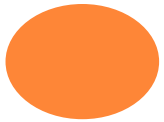
Decision

- If Chi square is in the critical region, we can reject H0 and accept H1 with 95% of confidence
- If Chi square is not in the critical region, we cannot reject H0 and cannot demonstrate H1

- Chi square = 37.2 in [3.84, +∞) , we can reject H0 and with 95% of confidence we accept H1
- With 95% of confidence there is association between smoking and lung cancer

CHI SQUARE USING STATISTICAL SOFTWARE

- **If you use a statistical software to apply Chi square test, you can make the decision by interpreting p value**
- **If p<=0,05 you can reject H0 and accept H1 with 95% of confidence**
- **If p>0,05 you cannot reject H0**

- **For our example p=0,00000000105**

Thank you !